STUDY GUIDE:

Module 1: The Development of Place Value

In this module we shall see how people first learned to count.
The most primitive way was to draw a picture of the object
being counted and to reproduce the picture once for every object that was
present.

Later, a simple mark (called a tally mark) was used to replace the
picture of the object being counted.

But as the number of objects being counted became greater and greater,
newer and better innovations had to be made so that we could recognize the
meaning of the numbers we were talking about.

In a step by step manner going from tally marks to Roman numerals; from
Roman numerals to the abacus; and from the abacus to our present place value
system, the saga of modern counting is unfolded.

The module ends with the message that the search for knowledge is
on-going and that no matter how advanced we become, new problems always
seem to manage to arise. In this way, we find that many of the problems
that bothered ancient peoples come back to haunt us even in the present
century.

Step 1:

> View Videotape Lecture #1

Step 2:

> Read Module 1 of the text.

Step 3:

> When you feel you understand the material presented in Steps 1
> and 2, complete the following "Check-The-Main-Ideas" Self-quiz
> by correctly filling in each blank.

Check the Main Ideas:

Arithmetic, like any other language uses symbols.
Symbols that represent numbers are called _____.   _numerals_
In the earliest sign languages it was clear that
people viewed numbers as _____ rather than as nouns.   _adjectives_
For example, to represent three horses, people would
draw _____ horses. And if they wanted to represent   _three (3)_
three people they would draw _____ people.   _three (3)_

Later they realized that they could just as
easily draw three _____ whether they were counting   _tally marks_
people or horses. For example, twelve sheep would
be represented as _____. But after awhile it | | | | | | | | | | | |
became difficult to distinguish one large number of
sheep from another large number of sheep. So people
invented special symbols or _____ to stand for   _numerals_
"ten", "hundred", and "thousand". The Romans used
___ to stand for "ten"; C to stand for _____ and   _X;_ _"hundred"_
M to stand for _____ . They also used ___ to   _"thousand";_ _I_
stand for "one". "one", "ten", "hundred" and
"thousand" are called _____ of ten.   _powers_

The problem with Roman numerals is that they
require a different letter of the alphabet for
each _____. To help avoid this problem   _power of ten_
people drew vertical lines in a horizontal row.
Each line represented a different _____.   _power of then_

For example if a marker was placed on the line furthest

to the right it stood for 1 _____. But if it was placed   *one*

on the next line to the left, the same marker stood

for 1 _____.   If no marker appeared on, say, the   *ten*

hundreds-line, it meant that there were ____ hundreds.   *no*

  To avoid having to draw lines, people invented

special numerals; 0,1,2,3,4,5,6,7,8, and 9; that

were called _____. The digit 0 was called a   *digits*

_____. For example by writing 407, the 0   *place-holder*

told us that there were no _____. 0 wasn't   *tens*

necessary in Roman numerals because the absence of

____'s told us there were no tens. To help make   *X*

it even easier to read and represent large numbers,

the digits were grouped from right-to-left in clusters

of _____. The first group named the number of _____.   *three; units*

The next group named the number of _____; and the   *thousands*

next group named the number of _____. So the   *millions*

numeral 435,856,207 meant 207 _____, 856 _____,   *units; thousands*

and _____ million. In this situation 0 was still   *435*

a place holder. For example 345,000,234 meant that

we had 345 _____. But if we wrote 345,234 it   *million(s)*

meant that we had 345 _____.   *thousands(s)*

    _____-notation was introduced to handle situations   *Scientific*

in which a great many digits appeared. To indicate that

we had a 1 followed by 37 zeroes we would write _____. 10^{37}

Conversely, if we saw 10^{37} it would tell us that we

had the numeral consisting of a _____ followed by   *1*

_____ 0's.   *thirty seven (37)*

Step 4:

Do the Mastery Review.

Mastery Review

1. Use tally marks to show how a shepherd   1. _____
    might keep track of the fact that he had
    nine sheep.

2. How would the shepherd have indicated that   2. _____
    he had thirteen sheep?

3. Using Roman numerals, how would the shepherd   3. _____
    have indicated that he had thirteen sheep?

4. Using no more than nine each of the X's and   4. _____
    I's, use Roman numerals to represent the number
    forty three (43).

5. How would we express the number three thousand   5. _____
    two hundred fourteen (3,214) using Roman numerals?

6. What number is shown on the sand-reckoner below? 6. _____

7. In the place value numeral 567 what noun does   7. _____
    5 modify?

8. In the place value numeral 675 what noun does   8. _____
    5 modify?

9. What number is named by CCCI?   9. _____

10. What number is named by: 10. _____

11. What number is named by 301?   11. _____

12. What number is named by 310?   12. _____

13. What number is named by 6540?   13. _____

14. How do we read the number: 14. _____
        6,403,697,492,184   ?
                                                         _____

                                                         _____

15. What number is named by 4,000,000,000 ?   15. _____

16. What number is named by 4,700,000 ?   16. _____

17. What number is named by 4,070,000 ?   17. _____

18. Write nine hundred sixty three as a place value numeral.   18. _____

19. Write nine hundred sixty three thousand as a place value numeral.   19. _____

20. Write nine hundred sixty three billion four hundred three million eight hundred thirty two thousand four hundred twelve as a place value numeral.   20. _____

21. Write fifteen billion four hundred thousand as a place value numeral.   21. _____

22. Write 100,000,000,000,000,000,000,000 in scientific notation.   22. _____

23. Write 100,000,000,000,000,000,000,000,000 in scientific notation.   23. _____

24. Write 10^{16} as a place value numeral.   24. _____

Answers:

1. ||||||||| 2. |||||||||||||| 3. *XIII*   4. *XXXXIII*   5. *MMMCCXIIII*

6. *three hundred fourteen (314)*   7. *hundred(s)*   8. *one(s)*

9. *three hundred one*   10. *three hundred one*   11. *three hundred one*

12. *three hundred ten*   13. *six thousand five hundred forty*

14. *6 trillion 403 billion 697 million 492 thousand 184 (units)*

15. *4 billion*   16. *4 million 700 thousand*   17. *4 million 70 thousand*

18. *963*   19. *963,000*   20. *963,403,832,412*   21. *15,000,400,000*
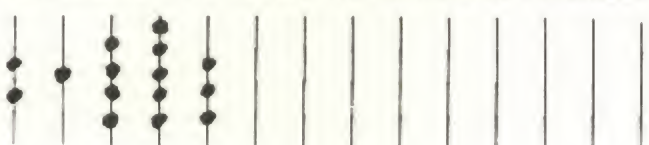
22. 10^{23} 23. 10^{26} 24. *10,000,000,000,000,000*

Do Self-Test 1: Form A

Self-Test 1: Form A

In problems 1 through 5; E stands for one, F stands for ten, G stands for a hundred, and H stands for a thousand.

1. What number is named by HHGFFFEEEE ?

1. _____

2. What number is named by HHHFFFF ?

2. _____

3. Which numeral names the greater number; GGGGGGFFFFF or H ?

3. _____

4. Without using more than nine of any one use E,F,G,and H to write the number four thousand three hundred fifty two.

4. _____

5. Rewrite GGGGGGGGGGFFFFFFFFFFFFFFFEEEEEEEEEEEEE so that none of the letter E,F,G,or H appears more than nine times.

5. _____

6. What number is shown on the sand reckoner below?

6. _____

_____

_____

_____

7. a. At $1 per object, what is the cost of 100 objects?

7 a. _____

b. At $1 per digit, what is the cost of the numeral, 100?

b. _____

8. Read the number represented by the numeral:

23,807,027,000,000,000 ?

8. _____

_____

9. Write as a place value numeral: eighty four quadrillion two hundred thirty seven billion.

9. _____

10. True or false: 10^4 is the number 10 followed by 4 zeroes.

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 1: Form A

1. 2,134 *(two thousand one hundred thirty four)*

2. 3,040 *(three thousand forty)*

3. H

4. HHHHGGGFFFFFEE

5. HFFFFEE

6. 21,453,000,000,000 *(twentyone trillion four hundred fifty three billion)*

7. a. $100

   b. $3

8. *twenty three quadrillion eight hundred seven trillion twenty seven billion*

9. 84,000,237,000,000,000

10. *False*

****

If you did each problem in Form A correctly, you may if you wish proceed

to the next module. Otherwise continue with Step 6.

****

Step 6:

    Study the Solutions to Self-Test 1: Form A, with special emphasis

    on any problems you failed to answer correctly.

1.

If we count the number of letters we have:

  two H's or 2 thousands

  one G or 1 hundred

  three F's or 3 tens

  four E's or 4 ones.

To get the adjectives (digits) that modify the powers of ten we have to count the number of times each letter appears

That is:

| thousands (H)	hundreds (G)	tens (F)	ones (E)
2	1	3	4

So the answer is 2,134

Alternative Solutions

(a) The symbol that names the power of ten is not the important thing. So, for example, if you feel more comfortable with Roman numerals, simply replace each E by an I, each F by an X, each G by a C and each H by an M. That is:

That is, both I and E are symbols for one; X and F are both symbols for ten etc.

```
HHGFFFEEEE
↓↓↓↓↓↓↓↓↓↓
MMCXXXIIII
```

(b) In terms of the sand reckoner, label the lines E,F,G,and H as shown below; placing a marker on each line for each time that letter appears.

    H   G   F   E
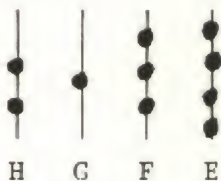
In this diagram the letter names the line while the number of markers give us the digit in each place.

Note the importance of order. Since the line furthest to the right names the ones-place, it must be labeled "E" because E represents "one".

2.

This is essentially the same problem as Problem 1
but emphasis on the place holder is introduced.

Again we count the number of times each letter
appears and we obtain:

three H's or 3 thousands

four F's or 4 tens.

Therefore:

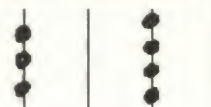| thousands (H)	hundreds (G)	tens (F)	ones (E)
3		4	

*If we now leave out the nouns and switch
to place value we must enter 0 as a place holder
in each column where we had none. That is:*

3	*0*	*4*	*0*

*In terms of the abacus or
sand reckoner, we have:*

3.

This problem tries to emphasize the importance
of viewing numbers as adjectives. For example, when
we say that 1 is less than 6 we assume that 1 and 6
are adjectives that modify the same noun. For example
1 *thousand* is *not* less that 6 *hundred*. In this problem
H means a thousand while GGGGGGFFFFF means 6 hundreds
and 5 tens, or 650. *Since one thousand is more than
six hundred fifty, H names the greater number.*

*GGGGGGFFFFF contains more
symbols than H but H names
the greater number. It's
analogous to 11 pennies be-
ing more coins than 1 quar-
er, but that 1 quarter has
greater purchasing power.*

4.

This is the "reverse" of the first three problems.
Now we start with our present numerals and construct
the "EFGH" numeral. To this end we observe that
four thousand three hundred fifty two means:

> 4 thousands or four H's
>
> 3 hundreds or three G's
>
> 5 tens or five F's
>
and
> 2 ones or two E's

That is, H still stands for a thousand, G stands for a hundred and so on.

Writing four H's, three G's, five F's, and
two E's we get:

HHHHGGGFFFFFEE

5.

The main aim of this problem is to emphasize
the power-of-ten property that ten of any denomination
is equal to one of the next greater denomination.

That is: ten 1's = 1 ten, ten 10's = 1 hundred, ten 100's =1 thousand etc.

In this case, we can exchange ten E's for an F;
ten F's for a G; ten G's for an H. Or, in other words,
we can "cross out" ten of any letter and replace them
by one more of the next letter. Therefore, starting
with GGGGGGGGGGFFFFFFFFFFFFFFFEEEEEEEEEE

(a) Cross out ten E's and annex an additional F:

GGGGGGGGGGFFFFFFFFFFFFFFF~~EEEEEEEEEE~~EE
F

(b) Cross out ten F's and annex another G:

GGGGGGGGGG~~FFFFFFFFFF~~FFF~~EEEEEEEEEE~~EE
G   F

If we had more than nine E's still left, we'd repeat step (a).

(c) Cross out ten G's and annex an H.

H ~~GGGGGGGGGG~~~~FFFFFFFFFF~~FFF~~EEEEEEEEEE~~EE
G   F

If we didn't have ten G's the process would end.

(cont)

5. (cont)

Now we simply copy over what hasn't been crossed
out to obtain HFFFFEE

*Note that we can "revisit" this problem
in more of a place-value format. Namely, we
have 9 hundreds, 13 tens, and 12 ones. In a
step-by-step way we can exchange ten of one
denomination for one of the next higher denom-
ination to obtain:*

thousands	hundreds	tens	ones
	9	13	12
	9	14	2
	10	4	2
1		4	2

which corresponds to 1 H, 4 F's, and 2 E's.

6.

This problem shows that even with the sand-
reckoner it is often difficult to keep track of
denominations. So in terms of thousands, millions,
and billions we have:

trillions   billions   millions   thousands   (units)

2 1   4 5 3

So the answer is 21 trillion 453 billion and if
we want to write the answer in place-value notation
we must use 0's to indicate the absence of millions,
thousands, and units. That is, the answer is:

21,453,000,000,000

*Notice that we have no need
for a place holder. We know
there are no hundreds be-
cause no G's appear.*

*That is, we have nine G's,
thirteen F's and twelve E's*

*Each row is a different way
of expressing the same num-
ber. In terms of money, if
you had nine $100-bills,
thirteen $10-bills and
twelve $1-bills, you'd have
a total of $1,042.*

*The commas were added for
convenience. Remember that
we start at the right and
"count off" in three's.*

*But as long as the lines
are present, we do not
need 0. The line itself
serves as the place holder.*

7.

The purpose of this problem is again to help you
distinguish between digits and the number represented
by the digits.

(a)

Here we are buying 100 objects. Since each object
costs $1, we are paying $1 a hundred times or a total
of $100.

(b)

Here we are buying only *three* objects. Namely the
numeral 100 consists of three digits--a 1 and two 0's.
Since each digit costs $1, the three digits combined
cost $3.

> *In other words, in place-value notation*
> *a hundred is represented by the 3-digit numeral*
> *100.*

For example, they could be
house numerals or jersey
numerals. Notice, in this,
case that 0 is indeed a dig-
it. You must pay $1 for it
the same as for the 1

8.

Here we have another example in which even using
place value there are many nouns to keep track of.
Recall how the nouns have been chosen:

$$\underset{\substack{\text{quad-}\\ \text{rillions}}}{\underline{2}\ \underline{3}}\ ,\ \underset{\text{trillions}}{\underline{8}\ \underline{0}\ \underline{7}},\ \underset{\text{billions}}{\underline{0}\ \underline{2}\ \underline{7}},\ \underset{\text{millions}}{\underline{0}\ \underline{0}\ \underline{0}},\ \underset{\text{thousands}}{\underline{0}\ \underline{0}\ \underline{0}},\ \underset{\text{units}}{\underline{0}\ \underline{0}\ \underline{0}}$$

Again notice in this system
of nouns, the biggest num-
erical adjective we need is
nine hundred ninety nine (999)

If we want to translate everything into words, we have
twenty three quadrillion eight hundred seven trillion
twenty seven billion.

9.

This is the "reverse" of the previous problem.
This time we "key" on the nouns "thousand", "million",
"billion" and so on. "Eighty Four" modifies "quadrillions"
and "two hundred thirty seven" modifies "billions". In
other words, we have:

84 quadrillion and 237 billions. Putting these in
their proper "places" we have:

_8_ _4_, _ _ _, _2_ _3_ _7_, _ _ _, _ _ _, _ _ _
quad-   trillions billions millions thousands units
rillions

Remember to put the 237 in the proper place. It modifies "billions"

Now replacing each dash by the place-holder 0 we have:

84,000,237,000,000,000

The dashes themselves could serve as the place-holders. The symbol 0 isn't importar The important thing is to hold the place.

10.

Nothing is more frustrating than a false statement
that is "almost" true. Yet in using scientific notation
it is important to understand that in 10^4, the 4 counts
the total number of 0's, *including the 0 in 10*. That
is, 10^4 is a 1 followed by four 0's--*not a 10* followed
by four 0's.

10 followed by four 0's would be 100000 and this is 10^5. 10^5 is 100,000 while 10^4 is 10,000. There is quite a difference between 10,000 and 100,000.

In general if n is any whole number 10^n names
the place value numeral that consists of a 1 followed
by n zeroes.

Step 7:

Do Self-Test 1: Form B

Self-Test 1: Form B

In problems 1 through 5; N stands for one, O stands
for ten, P stands for a hundred, and Q stands for
a thousand.

1. What number is named by QQQPPPPOON?   1. _____

2. What number is named by QQNNNN?   2. _____

3. Which numeral names the greater number: 3. _____
   P or OOOOOOONNNNN ?

4. Without using more than nine of any one   4. _____
   letter use N,O,P,and Q to write the the
   number five thousand six hundred thirty two.

5. Rewrite QQPPPPPPPPPPPPPOOOOOOOCOONNNNNNNNNNNNNNNN   5. _____
   so that none of the letters N, O, P, or Q
   appear more than nine times.

6. What number is shown on the sand-reckoner below?   6. _____

   _____

   _____
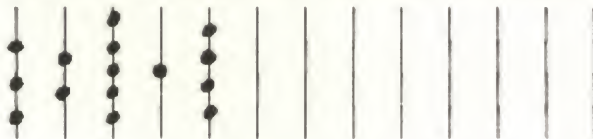
7. a. At $1 per object what is the cost of   7. a. _____
      300 objects?

   b. At $1 per digit, what is the cost of   b. _____
      the numeral 300?

8. Read the number represented by the numeral: 8. _____

   187,813,005,000,000,000

9. Write as a place value numeral: 9. _____

   Eight hundred four quadrillion five hundred
   thirty seven million.

10. True or False: 10^7 is 10 followed by 7 zeroes?   10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 1:   Form B

1. 3,421 (three thousand four hundred twenty one)

2. 2,004 (two thousand four)

3. P

4. QQQQQPPPPPPOOONN

5. QQQPPNNNN

6. 3,251,400,000,000 (three trillion two hundred fifty one billion four hundred million)

7. a. $300
    b. $3

8. one hundred eighty seven quadrillion eight hundred thirteen trillion five billion

9. 804,000,000,037,000,000

10. False

****

If you did each problem in Form B correctly, you may, if you wish,

proceed to the next module. Otherwise continue with Step 8.

****

Step 8:

View the solutions for Self-Test 1: Form B on Videotape Lecture 1S.

Pay special attention to the solutions of those problems for which

you failed to get the correct answers. _Feel free to rewind the tape_

at any time to restudy problems that give you difficulty.

*   *   *   *   *   *

Step 9:

Do Self-Test 1: Form C

In problems 1 through 5; e stands for a thousand,
f stands for a hundred, g stands for ten, and h stands
for one.

1. What number is named by hhgggfeeee ?   1. _____

2. What number is named by eeggg ?   2. _____

3. Which numeral names the greater number: 3. _____

    h or ggg ?

4. Without using any more than nine of each of   4. _____
    letters e,f,g,or h write the number six
    thousand five hundred forty three.

5. Rewrite eeffffffffffgggggggggghhhhhhhhhhhhh   5. _____
    so that none of the letters e,f,g,or h appear
    more than nine times.

6. What number is shown on the sand-reckoner below?   6. _____

                                                               _____

                                                               _____

                                                               _____

7. a. At $1 per digit, what is the cost of the
        numeral 400?   7. a. _____

    b. At $1 per object, what is the cost of   b. _____
        400 objects?

8. Read the number represented by the numeral: 8. _____

    46,830,050,000,000,000

9. Write as a place value numeral: 9. _____

    Seventy quadrillion eight hundred thirty
    nine thousand.

10. True or false: 10^9 is a 1 followed by 9 zeroes?   10. _____

            (ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 1: Form C

1. 4,132 *(four thousand one hundred thirty two)*

2. 2,030 *(two thousand thirty)*

3. *ggg*

4. *eeeeeeffffffgggghhh*

5. *eeeghhhh*

6. 23,120,000,000,000 *(twenty three trillion one hundred twenty billion)*

7. a. $3

   b. $400

8. *forty six quadrillion eight hundred thirty trillion fifty billion*

9. 70,000,000,000,839,000

10. *True*

****

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #1.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #2.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE YOU SHOULD
CONSULT A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

****